Calculus Warm Up #4- 1

Find the derivative using 2 different approaches. Which was easier?

$$f(x) = \frac{x^3 - 2x + 3}{\sqrt[3]{x}}$$

HW Questions: Review worksheet

1. $\lim_{x \to 3} \left(2x^2 - 3\right) =$

2. $\lim_{t \to 2} \frac{t^2 + t - 6}{t - 2} =$

3. $\lim_{\Delta x \to 0} \frac{\frac{1}{x + \Delta x} - \frac{1}{x}}{\Delta x} =$

4. $\lim_{w \to 0} \frac{\sqrt{w + 2} - \sqrt{2}}{w} =$

5. $\lim\limits_{x \to \left(\frac{1}{2}\right)^+} \frac{1}{2x-1} =$

6. Use the definition of the derivative to find y′:

$y = 2x^3 + x^2 - 3$

$$y' = \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^3 + (x+\Delta x)^2 - 3 - f(x)}{\Delta x}$$

7. $f(x) = \frac{x^2 - 2x + 3}{\sqrt{x}}$

$$f(x) = (x^2 - 2x + 3)(x^{-1/2})$$

$$f'(x) = (x^2 - 2x + 3)\left(-\frac{1}{2}x^{-3/2}\right) + (2x-2)x^{-1/2}$$

$$= \frac{-(x^2 - 2x + 3)}{2x^{3/2}} + \frac{2x-2}{x^{1/2}} \cdot \frac{2x}{2x}$$

8. $g(x)=(2x^2-1)(3x-2)$

9. $h(x)=\dfrac{2x-1}{3x+4}$

10. $y=2(-2x^2-3x+4)^4$

$y=8(-2x^2-3x+4)^3(-4x-3)$

11. $f(x)=\dfrac{2x-1}{\cancel{(x-1)^2}}$ $(2x-1)(x-1)^{-2}$

$f'(x)=(2x-1)(-2)(x-1)^{-3}(1)+2(x-1)^{-2}$

$$\frac{-4x+2}{(x-1)^3}+\frac{2}{(x-1)^2}\cdot\frac{x-1}{x-1}$$

12. $g(t) = 2t(t^2-4)^4$

Product Rule

$g'(t) = \underbrace{(2t)(4)(t^2-4)^3(2t)}\ +\ \underbrace{2(t^2-4)^4}$

$= 2(t^2-4)^3(8t^2 + t^2 - 4)$

$= 2(t^2-4)^3(9t^2-4)$

13. Find the tangent line of

$f(x) = 2x^2 - 3x + 1$ at $(1,0)$

14. Find all points of horizontal tangency of the following function,

$f(x) = 2x^3 - 3x^2 - 12x$

$\rightarrow f'(x) = 0$

$f'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

$x = 2, -1$

$f(2) =$

$f(-1) =$

$(-1,\ \ )$

$(2,\ \ )$

15. Find the second derivative of $f(x) = \dfrac{2x-1}{x-1}$

$f''(x) = \dfrac{2}{(x-1)^3}$

3.6

-Implicit and explicit functions

-Implicit differentiation

Explicit function form: $y = f(x)$

Examples: $y = 2x + 1$

$s = -16t^2$

$u = 2w - w^2$

One variable is explicitly given in terms of the other variable

Implicit function form:

Example: $xy = 1$

One variable is not given explicitly in terms of the other but rather implied by the equation.

Solve for y if you can!

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Solve for y if you can!

Then differentiate.

$xy = 1$

$y = \frac{1}{x}$

$y = x^{-1}$

$\frac{dy}{dx} = -1(x^{-2})$

$\frac{dy}{dx} = -\frac{1}{x^2}$

This method won't work if we can't solve for y!

What about $\frac{d}{dx}[y^2]$?

Apply the Chain Rule: $2y \cdot \frac{dy}{dx}$

Find the derivative, $\frac{d}{dx}$:

1. $y = 3x^2$

$\frac{dy}{dx} = 6x$

2. $y = x + 3y$

$-3y \quad -3y$

$-2y = x$

$y = \frac{x}{-2}$

$\frac{dy}{dx} = -\frac{1}{2}$

3. $y = 2x + y^2$

$\frac{dy}{dx} = 2 + 2y\frac{dy}{dx}$

$\frac{dy}{dx} - 2y\frac{dy}{dx} = 2$

$\frac{dy}{dx}(1 - 2y) = 2$

$\frac{dy}{dx} = \frac{2}{1-2y}$

Implicit Differentiation:

Finding $\frac{dy}{dx}$ for equations in x & y:

1. Differentiate both sides of the equation with respect to x
2. Collect all terms involving $\boxed{dy/dx}$ on the left side of the equation and move all other terms to the right side of the equation
3. Factor dy/dx out of the left side of the equation
4. Solve for dy/dx by dividing both sides of the equation by the left-hand factor that does not contain dy/dx

Find $\frac{dy}{dx}$:

$$x^2 + y^2 = 5xy$$

← Need product Rule!

$$2x + 2y\frac{dy}{dx} = 5x\frac{dy}{dx} + 5y$$

$$2y\frac{dy}{dx} - 5x\frac{dy}{dx} = 5y - 2x$$

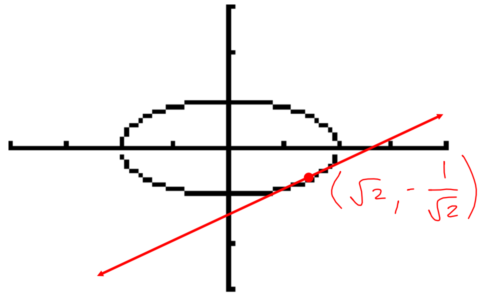
$$\frac{dy}{dx}(2y - 5x) = 5y - 2x$$

$$\frac{dy}{dx} = \frac{5y - 2x}{2y - 5x}$$

Find the tangent line to the graph of

$x^2 + 4y^2 = 4$ at the point $(\sqrt{2}, -\frac{1}{\sqrt{2}})$

slope $= \frac{dy}{dx}$



$2x + 8y\frac{dy}{dx} = 0$

$-2x \qquad -2x$

$\frac{8y\frac{dy}{dx}}{8y} = \frac{-2x}{8y}$

$\frac{dy}{dx} = -\frac{x}{4y}$

$m = \frac{1}{2}$

$y + \frac{1}{\sqrt{2}} = \frac{1}{2}(x - \sqrt{2})$

Find y″ :

$x^2 + y^2 = 25$

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(1) Find first derivative

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$2y\frac{dy}{dx} = -2x$

$\boxed{\frac{dy}{dx} = -\frac{x}{y}}$

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(2) Rewrite: $y' = -x(y^{-1})$

(I like product rule ☺)

$y'' = -x\left(-1y^{-2}\frac{dy}{dx}\right) + (-1)y^{-1}$

$y'' = \frac{x}{y^2} \cdot \frac{dy}{dx} - \frac{1}{y}$

Find y″ :

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① Find first derivative

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③ Replace $\frac{dy}{dx}$ with $-\frac{x}{y}$

$y'' = \frac{x}{y^2}\cdot\frac{-x}{y} - \frac{1}{y}\cdot\frac{y^2}{y^2}$

$y'' = -\frac{x^2+y^2}{y^3}$

But we can do more!

$x^2+y^2=$

Find y″ :

$x^2 + y^2 = 25$

$$y'' = \frac{x}{y^2} \cdot \frac{-x}{y} - \frac{1}{y} \cdot \frac{y^2}{y^2}$$

$$y'' = -\frac{x^2 + y^2}{y^3}$$

$$\frac{d^2y}{dy^2} = y''$$

But we can do more!

$x^2 + y^2 = 25$

So:

$$\boxed{y'' = -\frac{25}{y^3}}$$

HW: p. 141 # 1 - 17 odd,

\# 25 - 33 odd

Get organized for HW quiz tomorrow:

pgs. 119, 111, 127, 134, tan rev. ws